Life-cycle asset allocation and unemployment risk

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Motivation

- Unemployment may become a trap

  Percentage of Unemployment by Duration
  
<table>
<thead>
<tr>
<th></th>
<th>&gt;27 weeks</th>
<th>&gt;52 weeks</th>
<th>&gt;99 weeks</th>
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<tbody>
<tr>
<td>2011</td>
<td>44%</td>
<td>31%</td>
<td>16%</td>
</tr>
<tr>
<td>2014</td>
<td>33.5%</td>
<td>22%</td>
<td>11%</td>
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- Re-employability only 11% of the long-term unemployed in any given month found full-time work a year later. (Krueger, Cramer and Cho, 2014)

- Earning losses large and persistent (about 25%), Jacobson, LaLonde, and Sullivan (JLS, 1993)

- Human capital depreciation increases with unemployment duration (Keane and Wolpin, 1997; Arulampalam, 2001)

- In Life cycle models, unemployment has only transitory effects on income
Previous Literature

‘Standard’ Life cycle models (Cocco et al., 2005; Bagliano et al., 2014)
- no unemployment
- permanent and transitory labor income shocks

Life cycle models with unemployment risk
- transitory effects on labor income with both short (Carroll, 2002) and long (Bremus and Kuzin, 2014) run unemployment unemployment affects earnings (UB) over the unemployment spell (transitory effect), no impact on human capital

Labor supply flexibility (Gomes, Kotlikoff and Viceira, 2008)
- labor supply flexibility to hedge poor market performance
Life-Cycle Stock Shares
The present paper

- **Unemployment modeling**
  - Both short and long run unemployment: three-state Markov chain (as in Bremus and Kuzin, 2014)
  - Unemployment benefits (UB) during unemployment spell
  - Explicit human capital erosion: cut in the permanent component of labor income, to capture diminished future income prospects
  - Optimal stock share may be flat or moderately increasing in wealth and age
The Model

- Partial equilibrium
- Time is discrete
- Finite horizon with uncertain lifespan
- Standard expected utility preferences

\[
\frac{C_{it0}^{1-\gamma}}{1-\gamma} + E_{t0} \left[ \sum_{j=1}^{T} \beta^j \left( \prod_{k=0}^{j-2} p_{t0+k} \right) \left( p_{t0+j} \frac{C_{it0+j}^{1-\gamma}}{1-\gamma} + (1 - p_{t0+j}) b \frac{(X_{it0+j}/b)^{1-\gamma}}{1-\gamma} \right) \right]
\]

- \( C_{it} \) level of consumption at time \( t \)
- \( X_{it} \) wealth the investor leaves as a bequest,
- \( b \geq 0 \) strength of the bequest motive,
- \( \beta < 1 \) discount factor
- \( \gamma \) constant relative risk aversion parameter.
The Model

Labor Market Dynamics and Income

Transition matrix

\[
\begin{bmatrix}
\pi_{e,e} & \pi_{e,u_1} & \pi_{e,u_1} \\
\pi_{u_1,e} & \pi_{u_1,u_1} & \pi_{u_1,u_2} \\
\pi_{u_2,e} & \pi_{u_2,u_1} & \pi_{u_2,u_2}
\end{bmatrix}
= \begin{bmatrix}
\pi_{e,e} & 1 - \pi_{e,e} & 0 \\
\pi_{u_1,e} & 0 & 1 - \pi_{u_1,e} \\
1 & 0 & 0
\end{bmatrix}
\]

Labor income process

\[Y_{it} = H_{it} N_{it}, \quad t_0 \leq t \leq t_0 + K\] (1)

- \(H_{it} = (F(t, Z_{it}) P_{it})\) permanent income component
- \(F(t, Z_{it}) \equiv F_{it}\) deterministic trend component
- \(\log P_{it} = \log P_{it-1} + \omega_{it}\) (2)

stochastic permanent component

- \(\omega_{it} \sim N(0, \sigma^2_\omega)\)
- \(\log(N_{it}) \sim N(0, \sigma^2_\epsilon)\)
The Model

Labor Market Dynamics and Income

- Labor income depends on past working history

\[ H_{it} = \begin{cases} 
H_{it} & \text{if } s_t = e \text{ and } s_{t-1} = e \\
(1 - \psi_1)H_{it-1} & \text{if } s_t = e \text{ and } s_{t-1} = u_1 \\
(1 - \psi_2)H_{it-2} & \text{if } s_t = e \text{ and } s_{t-1} = u_2 
\end{cases} \quad t = t_0, \ldots, t_0 + K \tag{3} \]

- with $\psi_2 > \psi_1$

- Unemployment insurance:

\[ Y_{it} = \begin{cases} 
\xi_1 H_{it-1} & \text{if } s_t = u_1 \text{ and } s_{t-1} = e \\
0 & \text{if } s_t = u_2 \text{ and } s_{t-1} = u_1 \text{ and } s_{t-2} = e 
\end{cases} \quad t = t_0, \ldots, t_0 + K \tag{4} \]

- Income received during retirement

\[ Y_{it} = \lambda F \left( t, Z_{it_{0+K}} \right) P_{it_{0+K}} \quad t_0 + K < t \leq T \tag{5} \]

- $\lambda$ of the permanent component of labor income in the last working year
The Model

- Financial assets
  - one period risk free asset with return $R_t^m$
  - risky stock with return
    \[
    \tilde{R}_t^s = R_t^m + \mu^s + \nu_t^s \tag{6}
    \]
    with
    \[
    \nu_t^s \sim N(0, \sigma^2_s)
    \]
  - no borrowing/short sale constraints
- portfolio return
  \[
  R_{it}^P = \alpha_{it}^s R_t^s + (1 - \alpha_{it}^s) R^f \tag{7}
  \]
- Cash on hand
  \[
  X_{it+1} = (X_{it} - C_{it}) R_{it}^P + Y_{it+1} \tag{8}
  \]
  $\alpha_{it}^s$ denotes the shares of the investor’s portfolio invested in stocks
The Model

**Individual’s optimal program**

\[
\max_{\{c_{it}\}_{t_0}^{T-1},\{\alpha_{it}\}_{t_0}^{T-1}} \left( \frac{c_{it_0}^{1-\gamma}}{1-\gamma} + \left( \sum_{j=1}^{T} \beta^{j} \left( \prod_{k=0}^{j-2} p_{t_0+k} \right) \left( \frac{c_{it_0+j}^{1-\gamma}}{1-\gamma} \right) + (1 - p_{t_0+j}) b \left( \frac{X_{it_0+j}/b}{1-\gamma} \right) \right) \right) \]  

\[\text{s.t. } X_{it+1} = (X_{it} - C_{it}) (\alpha_{it}^s R_t^s + (1 - \alpha_{it}^s) R^f) + Y_{it+1}\]  

**Dynamic Programming Form**

\[
V_{it} (X_{it}, P_{it}, s_{it}) = \max_{\{c_{it}\}_{t_0}^{T-1},\{\alpha_{it}\}_{t_0}^{T-1}} \left( \frac{c_{it}^{1-\gamma}}{1-\gamma} + \beta E_t \left[ p_t V_{it+1} (X_{it+1}, P_{it+1}, s_{it+1}) \right] \right) 
+ (1 - p_t) b \left( \frac{X_{it+1}/b}{1-\gamma} \right) \]  

(11)
The Model

Value function

\[
V_{it} (X_{it}, P_{it}, s_{it}) = \max_{\{C_{it}\}_{t=0}^{T-1}, \{\alpha_{it}^s\}_{t=0}^{T-1}} \left( \frac{C_{it}^{1-\gamma}}{1 - \gamma} \right) \\
+ \beta \left[ p_t \sum_{s_{it+1} = e_1, u_2} \pi (s_{it+1} | s_{it}) \hat{E}_t V_{it+1} (X_{it+1}, P_{it+1}, s_{it+1}) \right] \\
+ (1 - p_t) b \sum_{s_{it+1} = e_1, u_2} \pi (s_{it+1} | s_{it}) \frac{(X_{it+1} / b)^{1-\gamma}}{1 - \gamma}
\]
The Model

Value function in each possible labor market state

\[
V(X_{it}, P_{it}, e) = u(C_{it}) + \beta p_t
\]

\[
\begin{cases}
V(X_{it+1}, P_{it+1}, e) \\ \\
\text{with prob. } \pi_{e,e}
\end{cases}
\]

with \( P_{it+1} = P_{it} e^{\omega_{it+1}} \) and

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it+1} P_{it+1} e^{\varepsilon_{it+1}}
\]

\[
\begin{cases}
V(X_{it+1}, P_{it+1}, u_1) \\ \\
\text{with prob. } \pi_{e,u_1}
\end{cases}
\]

with \( P_{it+1} = (1 - \Psi_1) P_{it} \) and

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + \xi_1 F_{it} P_{it}
\]

\[
\begin{cases}
V(X_{it+1}, P_{it+1}, e) \\ \\
\text{with prob. } \pi_{u_1,e}
\end{cases}
\]

with \( P_{it+1} = (1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} \) and

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it-1} (1 - \Psi_1) P_{it-1} e^{\varepsilon_{it+1}}
\]

\[
\begin{cases}
V(X_{it+1}, P_{it+1}, u_2) \\ \\
\text{with prob. } \pi_{u_1,u_2}
\end{cases}
\]

with \( P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} \) and

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + \xi_2 F_{it-1} (1 - \Psi_2)(1 - \Psi_1) P_{it-2} e^{\varepsilon_{it+1}}
\]

\[
\begin{cases}
V(X_{it+1}, P_{it+1}, e) \\ \\
\text{with prob. } \pi_{u_2,e}
\end{cases}
\]

with \( P_{it+1} = (1 - \Psi_2)(1 - \Psi_1) P_{it-1} e^{\omega_{it+1}} \) and

\[
X_{it+1} = (X_{it} - C_{it}) R_{it}^p + F_{it-2} (1 - \Psi_2)(1 - \Psi_1) P_{it-2} e^{\varepsilon_{it+1}}
\]
Benchmark parameters

- Risk aversion coefficient $\gamma = 5$
- Discount factor $\beta = 0.96$
- Bequest motive $b = 2.5$
- Equity premium set at $4\%$
- Return on the risk free asset $2\%$
- Social security replacement rate $\lambda = 0.68$
- Transition matrix between labor market states

\[
\begin{bmatrix}
\pi_{e,e} & 1 - \pi_{e,e} & 0 \\
\pi_{u_1,e} & 0 & 1 - \pi_{u_1,e} \\
1 & 0 & 0 \\
\end{bmatrix} = \begin{bmatrix}
0.96 & 0.04 & 0 \\
0.95 & 0 & 0.05 \\
1 & 0 & 0 \\
\end{bmatrix}
\]

- Unemployment insurance $\xi_1 = 0.3$, $\xi_2 = 0$.
- Human capital erosion during unemployment
  - low: $\psi_1 = 0.2$, $\psi_2 = 0.4$
  - high: $\psi_1 = 0.2$, $\psi_2 = 0.9$
- The implied labor income process is not significantly different from the case without unemployment risk
Optimal stock shares - standard model without unemployment risk

The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in the baseline case, without unemployment risk. The policies are plotted for selected ages 25 and 40 years old.
Optimal stock shares- all models

The figure shows the portfolio rules for stocks as a function of cash on hand for a medium level of the stochastic permanent labor income component in case of unemployment risk. The parameters governing the human capital erosion during short-term and long-term unemployment spells are equal to 0.2(\psi_1) and 0.4(\psi_2), respectively. The policies are plotted for selected ages: 20 and 40.
The figure displays the mean simulated stock profiles for individuals of age 20 to 100. Risk aversion $\gamma = 5$, social security replacement ratio = 0.68. Three cases are considered. Calibration 1 the benchmark model without unemployment risk. Calibration 2 with unemployment risk and moderate human capital erosion: $0.2(\psi_1)$ and $0.4(\psi_2)$. Calibration 3 with unemployment risk and strong human capital erosion: $0.2(\psi_1)$ and $0.9(\psi_2)$. 

Optimal Life-Cycle Profiles

![Optimal life cycle profiles](image-url)
Conclusions

▶ We model long-term unemployment risk embedding observed consequences of long term unemployment.

▶ An even small probability of experiencing high human capital erosion is able to generate optimal conditional stock shares in line with those observed in the data.

▶ These differ from those embedded in most common Target Date Retirement Funds, that may generate large welfare losses
Extensions

- participation costs (Gomes and Michaelides, 2009)
- two risky assets (Bagliano, Fugazza and Nicodano, 2014)