Networks in risk spillovers: a multivariate GARCH perspective

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**Motivation**

**Explaining volatility spillover through information on cross-country exposure**

- Use cross-country exposure to identify a dynamic network structure
- Introduce them into a variance model to capture spillovers
- Generalize a variance model to allow for spillover driven by a time-varying network
- Show how to use the model for spillover analysis, variance forecasting and policy intervention
Spatial proximity and financial closeness

- In Spatial Econometrics subjects of the analysis are neighbors in a physical sense, and distance between subjects is measured by means of geographical distance.
- From a financial viewpoint a neighboring relation (closeness, proximity) is a more elusive concept.
- ... that cannot be translated into a physical measure (Fernandez-Aviles et al., 2012).
- Possible solutions:
  - subjects (companies) being in the same economic sector (Caporin and Paruolo, 2015)
  - causality relations (Billio et al., 2015)
  - cross-country exposure (this study)
Financial proximity and networks

- In spatial econometrics distances are collected into spatial weight matrices $W$ which are symmetric (before row normalization) and positive valued.
- In financial applications $W$ need not to be symmetric (despite we assume it is positive valued).
- $W$ matrices are also equivalent to adjacency matrices associated with a weighted directed network, therefore, from a financial viewpoint, proximity relations define networks across subjects, and, a given network existing across subjects allows structuring a spatial weight matrix.
Proximity matrices for model construction

- Caporin and Paruolo (2015) introduce a more general proximity matrix

\[ P = \rho_1 I + \rho_2 W \]

- Motivations: \( P \) is a sparse matrix driven by few unknown parameters; \( P \) structure depends on economically relevant relations
- Easily generalized to allow for heterogenous reaction replacing \( \rho_j \) with a diagonal matrix
- **This study:** allows for dynamic in \( W \); allows for left and right multiplication to recover different interpretations (contributing to/receiving from a network)

\[ P_L = \text{diag} (\rho_0) I + \text{diag} (\rho_{1,L}) W_t \]
\[ P_R = \text{diag} (\rho_0) I + W_t \text{diag} (\rho_{1,R}) \]
Proximity matrices for covariance models

- Covariance models (GARCH/DCC-type, Stochastic Volatility, Realized Covariance/Correlation) suffer for the curse of dimensionality challenging their use with large cross-sectional dimension for spillover analyses.
- Intuition: use proximity matrices to structure the model parameters and make parameters number linear in the cross-sectional dimension.
- Intuition: dynamic proximity matrices induce dynamic in the parameters at limited cost (they are assumed to be known).
- We focus on BEKK-type specifications as we focus on covariance dynamic and for the availability of asymptotic theory.
BEKK and the curse of dimensionality

**BEKK model**

Given a vector $y_t$ of $n$ cross-sectional observations at time $t$ define $u_t = y_t - \bar{y}$ with $\bar{y}$ being the vector of sample means, we have:

$$u_t = \frac{1}{\sqrt{2}} \epsilon_t \quad \epsilon_t \sim \mathcal{N}(0, I_n), \quad t = 1, \ldots, T$$

$$\Sigma_t = CC' + A u_{t-1} u'_{t-1} A' + B \Sigma_{t-1} B'$$

where $A, B$ and $C$ are $n \times n$ matrices and $\Sigma_t^{1/2}$ is the Cholesky decomposition of $\Sigma_t$.

- Computationally demanding with even moderate cross-sectional dimension ($n > 3$) unfeasible for $n > 10$ ($n = 10 \Rightarrow 255$ parameters)
- Usually restriction on $A$ and $B$ are imposed; typical example $A, B$ are diagonal (D-BEKK) but we fully loose volatility spillover
Spatial BEKK (S-BEKK)

A and $B$ (BEKK parameter matrices) are proximity matrices and depend on an exogenous sequence of weights matrices $W_t$

$$\Sigma_t = CC' + A(W_t) u_{t-1} u_{t-1}' A(W_t)' + B(W_t) \Sigma_{t-1} B(W_t)'$$

- Nests the D-BEKK model but allows cross spillovers from different elements of $\Sigma_t$
- $A$ and $B$ are time varying due to the time variation of $W_t$
- The total number of parameters is linear in the cross-sectional dimension
Spatial BEKK (S-BEKK)

Extend proximity matrices with left and right multiplication

- $A$ and $B$ proximity matrices with right and left multiplication specification
  
  $A_L(W_t) = \text{diag}(a_{0L}) I + \text{diag}(a_{1L}) W_t$
  
  $B_L(W_t) = \text{diag}(b_{0L}) I + \text{diag}(b_{1L}) W_t$
  
  $A_R(W_t) = I \text{diag}(a_{0R}) + W_t \text{diag}(a_{1R})$
  
  $B_R(W_t) = I \text{diag}(b_{0R}) + W_t \text{diag}(b_{1R})$

- $W_t$ is the time dependent foreign claims network

- Left and right multiplication models allows focusing on different aspect of risk propagation

- With right multiplication the parameters multiply the source of risk: focus on risk spreaders entities

- With left multiplication the parameters multiply the recipient of risk: focus on risk receivers entities
Left and Right Indirect Effect

Extend notion of direct and indirect effect of shock diffusions

- LeSage and Pace (2014) SEM

\[ \nu_t = (I_n + \theta W) u_t = \nu^0_t + \nu^1_t \]
\[ \nu^0_{i,t} = [I_n u_t]_i = u_{i,t} \quad \text{Direct Effect} \]
\[ \nu^1_{i,t} = [\theta Wu_t]_i = [W \theta u_t]_i = \theta \sum_{j=1}^{n} \omega_{i,j} u_{j,t} \quad \text{Indirect Effect} \]

- Different Left (Risk Receivers), Right (Risk Spreaders), ARCH (Shock Response) and GARCH (Persistence) Indirect effects due to the non-commutativity of diagonal parameters and network matrices

\[ \nu^1_{L,i,t} = [A_{1,L} Wu_t]_i = a_{1,L,i} \sum_{j=1}^{n} \omega_{i,j} u_{j,t} \]
\[ m^1_{R,i,t} = [WB_{1,R} u_t]_i = \sum_{j=1}^{n} \omega_{i,j} b_{1,R,j} u_{j,t} \]
Model estimation

- Standard BEKK model asymptotic properties
  - consistency, Jeantheau (1988), under the existence of sixth-order moments
  - asymptotic normality, Comte and Lieberman (2003), under the existence of eighth-order moments, and Hafner and Preminger (2009) for VECH under the existence of sixth-order moments

- In our case we ensure ergodicity and stationarity following Boussama, Fuchs and Stelzer (2011) and constrain the maximum spectral radius

\[
\max_{t \in [1, T]} \rho (A_M(W_t) \otimes A_M(W_t) + B_M(W_t) \otimes B_M(W_t)) < 1
\]

- ...and recover consistency and asymptotic normality assuming the existence of the sixth-order moments
Inference-based networks

- Intuition: merge cross-holding networks and estimated parameters to recover a new network combining observed links (and their size) with statistically relevant spillovers.

- Filtered (inference-based) network (element $i, j$ with $i \neq j$)

\[
[W_{A,L}]_{t,i,j} = \hat{a}_{1,L,i} w_{t,i,j} \times (1 - Pval(\hat{a}_{1,L,i}))
\]

- Four different inference-based networks obtained from ARCH and GARCH coefficient matrices for both the Left and Right multiplication cases.
Inference-based networks

- Four different interpretations to the filtered networks
  - $\mathcal{W}^A_L$: focus on risk receivers entities and response to shocks
  - $\mathcal{W}^B_L$: focus on risk receivers entities and persistence of shocks
  - $\mathcal{W}^A_R$: focus on risk spreaders entities and response to shocks
  - $\mathcal{W}^B_R$: focus on risk spreaders entities and persistence of shocks

- Filtered networks allows analyzing entities links from a different viewpoint
## Variance decomposition

<table>
<thead>
<tr>
<th></th>
<th>Shock response (ARCH)</th>
<th>Persistence (GARCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costant</td>
<td>$[CC']_{i,j}$</td>
<td>$[\Omega^0,0]_{i,j}$</td>
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<tr>
<td>Direct</td>
<td>$v_{L,i,t-1}^0 v_{L,j,t-1}^0$</td>
<td>$[\Omega^0,1]_{i,j}$</td>
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<tr>
<td>Indirect</td>
<td>$v_{L,i,t-1}^1 v_{L,j,t-1}^1$</td>
<td>$[\Omega^1,0]<em>{i,j} + [\Omega^0,1]</em>{i,j}$</td>
</tr>
<tr>
<td>Mixed</td>
<td>$v_{L,i,t-1}^1 v_{L,j,t-1}^0 + v_{L,i,t-1}^0 v_{L,j,t-1}^1$</td>
<td>$[\Omega^1,0]<em>{i,j} + [\Omega^0,1]</em>{i,j}$</td>
</tr>
</tbody>
</table>

- Where, for example:

  \[
  \begin{bmatrix}
  \Omega^1,1_{L,t-1}
  \end{bmatrix}_{i,j} = \text{Cov} \left( m_{L,i,t-1}^1, m_{L,j,t-1}^1 \mid I_{t-2}, W \right)
  \]

- At the portfolio level (suppressing time-dependence)

  \[
  \nu [w'y] = \sigma^2_{\text{Constant}} + \sigma^2_{\text{Direct}} + \sigma^2_{\text{Indirect}} + \sigma^2_{\text{Mixed}}
  \]

- Diversification benefits could come from each component
Multistep Forecast for Optimal Networks

- What is the optimal network that minimizes the system conditional variance? What are the (target) exposures that minimize the risk in the system?
- In the following we propose a forecast based criteria
- To compute multi-step-ahead forecasts we resort to a simulation approach
- First we computed filtered innovations (standardized residuals) as
\[ \hat{\epsilon}_t = \Sigma_t^{1/2} u_t \]
- Generate \( N_B \) stationary bootstrap \( h \)–step samples from \( \tilde{\epsilon}_t^B \) and then generate \( N_B \) simulated values for \( \Sigma_t \)
\[ \tilde{\epsilon}_t^B = \Sigma_t^{1/2} \epsilon_t^B \]
\[ \Sigma_t^B = CC' + A\tilde{\epsilon}_{t-1}^B \tilde{\epsilon}_{t-1}^B A' + B\Sigma_{t-1}^B B' \]
- We take as forecast the average, but even quantiles could be considered if we want to focus on low/high volatility level forecasts
\[ \Sigma_t^F = \frac{1}{N_B} \sum_{B=1}^{N_B} \Sigma_t^B \]
Optimal networks and target exposures

- Conditional on bootstrapped innovations $\tilde{\epsilon}_{T+l}^{[b]}$ with $b \in [1, \ldots, N_B]$ find the optimal network by minimizing the average h-step forecasted variance path of the EW index with respect to the network

$$\min_{\text{vec}W^*} \frac{1}{h} \sum_{l=1}^{h} \frac{1}{n} 1' \hat{\Sigma}_{T+l}^F (W^*) 1$$

$$\text{s.t. } 0 \leq [W^*]_{i,j} \leq 1 \text{ for } i, j = 1 \ldots n$$

$$\text{Tr} (W^*) = 0$$

- We compare the obtained optimal volatility proxy with the realized volatility proxy, to have an indication of variance reduction robust against model misspecification.

$$\text{Var} \left( \frac{1}{n} 1'y_{T+l}^* \bigg| \mathcal{I}_{T+l-1} \right) \approx \left( \frac{1}{n} 1'u_{T+l}^* \right)^2$$
Data description

- Changes in the ten-year sovereign bond yields
- Sample size January 2006 - December 2013, coherently with the cross country holding data
- Model daily data with network evolving at a lower time scale, quarterly
- Networks estimated from BIS data with normalization associated with the total amount of bank exposures (thus not restricting the normalization to the analyzed countries)
Dependence Network from Foreign Claims

2012-Q2

Countries: [flags of different countries]
## Estimated Parameters

<table>
<thead>
<tr>
<th></th>
<th>$a_0$ D-BEKK</th>
<th>$b_0$ D-BEKK</th>
<th>$a_0$ S-BEKK L</th>
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* 10%  ** 5%  *** 1%
edge weight $\Rightarrow \hat{a}_{1,L,i} w_{t,i,j} \times (1 - Pval(\hat{a}_{1,L,i}))$

In red positive coefficients, in blue negative
Inference-based $B_L$ Networks

\[
\text{edge weight } \Rightarrow \hat{b}_{1,L,i} w_{t,i,j} \times \left(1 - Pval(\hat{b}_{1,L,i})\right)
\]

2006-Q1

In red positive coefficients, in blue negative
Inference-based $A_R$ Networks

edge weight $\Rightarrow \hat{a}_{1,R,i} w_{t,i,j} \times (1 - Pval(\hat{a}_{1,R,i}))$

In red positive coefficients, in blue negative
Inference-based $B_R$ Networks

\[ \text{edge weight } \Rightarrow \hat{b}_{1,R,i} w_{t,i,j} \times \left( 1 - \text{Pval} \left( \hat{b}_{1,R,i} \right) \right) \]

2006-Q1

In red positive coefficients, in blue negative
- Most comes from the direct contribution
- Negative contributions are possible and lead to diversification benefits
Variance decomposition Risk Spreaders (R)

- Most comes from the direct contribution
- Negative contributions are possible and lead to diversification benefits
Forecast one quarter ahead for quarter II of 2010
Recover the optimal target network exposure for both risk receivers and risk spreaders
Constrained optimization implies a redistribution of the claims among the considered countries
Unconstrained optimization means that the total amount of claims changes for each country
Optimal vs. Realized EW variance proxy

- Receivers Model Reconstructed Volatility Proxy, Estimation Sample Q1 2006 Q1 2010
- Receivers Model Constrained Reconstructed Volatility Proxy, Estimation Sample Q1 2006 Q1 2010
- Realized

- Spreaders Model Reconstructed Volatility Proxy, Estimation Sample Q1 2006 Q1 2010
- Spreaders Model Constrained Reconstructed Volatility Proxy, Estimation Sample Q1 2006 Q1 2010
- Realized
Investment Needed to Reach Target Exposures

<table>
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<th>IE</th>
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</tbody>
</table>

- Italy should have invested more across borders and Portugal should have received more investments from the other countries.
- In general, the sensible variance reduction we obtain from our calculation is implied by redistributions that are extreme and would be hard to enforce in a single quarter.
- But it is easy to include economically sound maximum redistribution constraints in the methodology.
Concluding remarks

- In the European sovereign bond markets part of spillover effects can be traced back to a physical claim channel: banks’ foreign exposures.
- Germany, Italy and, to a lesser extent, Greece are playing a central role in spreading risk.
- Ireland and Spain are the most susceptible receivers of spillover effects.
- Acting on these physical channels before the sovereign crisis, it would have been possible to have a clear risk mitigation outcome.

In progress

- Different Asset Classes (In particular equity)
- Inclusion of more countries
- Joint Left and Right multiplication model
- Introduce in the model more networks, jointly, to test and compare them.
Thank You!