Asset Allocation Strategies Based on Penalized Quantile Regression

Giovanni Bonaccolto\textsuperscript{1}

\textit{joint work with}
Massimiliano Caporin\textsuperscript{2} and Sandra Paterlini\textsuperscript{3}

9th Financial Risks International Forum

\textsuperscript{1}Department of Statistical Sciences, University of Padova, Italy
\textsuperscript{2}Department of Economics and Management, University of Padova, Italy
\textsuperscript{3}Department of Finance and Accounting, EBS University, Germany
Motivations

Aim: developing asset allocation strategies by combining quantile regression and regularization techniques
Motivations

Aim: developing asset allocation strategies by combining quantile regression and regularization techniques, with focus on:

- different performance indicators;
Motivations

Aim: developing asset allocation strategies by combining quantile regression and regularization techniques, with focus on:

- different performance indicators;
- in- and out-of-sample outcomes;
Motivations

Aim: developing asset allocation strategies by combining quantile regression and regularization techniques, with focus on:

- different performance indicators;
- in- and out-of-sample outcomes;
- stability and robustness of the solutions;
Motivations

Aim: developing asset allocation strategies by combining quantile regression and regularization techniques, with focus on:

- different performance indicators;
- in- and out-of-sample outcomes;
- stability and robustness of the solutions;
- high dimensionality.
OLS and Global Minimum Variance Portfolio

\[ \mathbf{R} = [R_1, \ldots, R_n] \]: vector of assets returns with covariance matrix \( \Sigma \);

\[ \mathbf{w} = [w_1, \ldots, w_n] \]: portfolio weights vector, \( \mathbf{w}' = 1 \) (budget constraint);

\[ R^*_i = R_n - R_i, \quad i = 1, \ldots, n - 1; \]

\[ R_p = R_n - w_1 R^*_1 - \ldots - w_n R^*_n - \ldots \quad (1) \]

\[ \text{GMVP}: \min_{\mathbf{w} \in \mathbb{R}^n} \mathbf{w}' \Sigma \mathbf{w} = \min_{(\mathbf{w} - n, \xi) \in \mathbb{R}^n} (\mathbf{w} - n, \xi) \]

\[ E[\mathbf{R}_n - w_1 R^*_1 - \ldots - w_n R^*_n - \ldots - \xi]^2 \quad (2) \]

where \( \mathbf{w} - n = [w_1, \ldots, w_n - 1] \), \( \xi \) is the intercept, see e.g. Fan, Zhang, and Yu (2012).
OLS and Global Minimum Variance Portfolio

- $\mathbf{R} = [R_1, \ldots, R_n]$: vector of assets returns with covariance matrix $\Sigma$;
- $\mathbf{w} = [w_1, \ldots, w_n]$: portfolio weights vector, $\mathbf{1}_n^T \mathbf{w} = 1$ (budget constraint);

\[ R^*_i = R_n - R_i, \quad i = 1, \ldots, n - 1; \]

\[ R_p = R_n - \mathbf{w}_1 R^*_1 - \cdots - \mathbf{w}_{n-1} R^*_{n-1}; \]

\[ \text{GMVP: } \min_{\mathbf{w} \in \mathbb{R}^n} \mathbf{w}^T \Sigma \mathbf{w} = \min_{(\mathbf{w} - n, \xi) \in \mathbb{R}^n} (\mathbf{w} - n, \xi) \]

\[ \left( \mathbb{E}[R_n - \mathbf{w}_1 R^*_1 - \cdots - \mathbf{w}_{n-1} R^*_{n-1} - \xi] \right)^2; \]

where $\mathbf{w} - n = [w_1, \ldots, w_{n-1}]$, $\xi$ is the intercept, see e.g. Fan et al. (2012).
OLS and Global Minimum Variance Portfolio

- \( \mathbf{R} = [R_1, \ldots, R_n] \): vector of assets returns with covariance matrix \( \Sigma \);

- \( \mathbf{w} = [w_1, \ldots, w_n] \): portfolio weights vector, \( \mathbf{1} \mathbf{w}' = 1 \) (budget constraint);

- \( R_i^* = R_n - R_i, \ i = 1, \ldots, n - 1 \);
OLS and Global Minimum Variance Portfolio

- $\mathbf{R} = [R_1, \ldots, R_n]$: vector of assets returns with covariance matrix $\Sigma$;

- $\mathbf{w} = [w_1, \ldots, w_n]$: portfolio weights vector, $\mathbf{1} \mathbf{w}' = 1$ (budget constraint);

- $R_i^* = R_n - R_i$, $i = 1, \ldots, n - 1$;

\[
R_p = R_n - w_1 R_1^* - \ldots - w_{n-1} R_{n-1}^*
\] (1)
OLS and Global Minimum Variance Portfolio

- \( \mathbf{R} = [R_1, \ldots, R_n] \): vector of assets returns with covariance matrix \( \Sigma \);

- \( \mathbf{w} = [w_1, \ldots, w_n] \): portfolio weights vector, \( \mathbf{1w}' = 1 \) (budget constraint);

- \( R^*_i = R_n - R_i, \ i = 1, \ldots, n - 1; \)

\[
R_p = R_n - w_1 R^*_1 - \ldots - w_{n-1} R^*_{n-1}
\] (1)

**GMVP:**

\[
\min_{\mathbf{w} \in \mathbb{R}^n} \mathbf{w} \Sigma \mathbf{w}' \quad = \quad \min_{(\mathbf{w}_{-n}, \xi) \in \mathbb{R}^n} \mathbb{E} \left[ R_n - w_1 R^*_1 - \ldots - w_{n-1} R^*_{n-1} - \xi \right]^2 \quad (2)
\]

where \( \mathbf{w}_{-n} = [w_1, \ldots, w_{n-1}] \), \( \xi \) is the intercept, see e.g. Fan et al. (2012).
**Choquet Expected Utility:**

\[
\mathbb{E}_\nu[u(R_p)] = \int_0^1 u \left( F_{R_p}^{-1}(\vartheta) \right) d\nu(\vartheta),
\]

(3)

\(u(\cdot)\): utility function (Hp monotone increasing), \(\nu(\cdot)\): distortion function, \(\vartheta\): probability level, \(F_{R_p}(r_p)\): distribution function of \(R_p\) at \(r_p\);

\[\alpha\text{-risk (coherent risk measure (Artzner, Delbaen, Eber, & Heath, 1999))},
\]

\[
\varrho_{\nu\alpha}(R_p) = -\int_0^1 F_{R_p}^{-1}(\vartheta) d\nu(\vartheta) = -\alpha^{-1} \int_0^\alpha F_{R_p}^{-1}(\vartheta) d\vartheta
\]

(4)
Pessimistic Asset Allocation and Choquet Utility

**Choquet Expected Utility:**

\[
\mathbb{E}_\nu[u(R_p)] = \int_0^1 u \left(F_{R_p}^{-1}(\vartheta)\right) d\nu(\vartheta),
\]

(3)

\(u(\cdot):\) utility function (Hp monotone increasing), \(\nu(\cdot):\) distortion function, \(\vartheta:\) probability level, \(F_{R_p}(r_p):\) distribution function of \(R_p\) at \(r_p;\)

let \(\alpha\) be a low probability level, if \(\nu_\alpha(\vartheta) = \min\{\vartheta/\alpha, 1\}\) (Bassett et al., 2004), then

\[
\mathbb{E}_{\nu_\alpha}[u(R_p)] = \alpha^{-1} \int_0^\alpha u \left(F_{R_p}^{-1}(\vartheta)\right) d\vartheta
\]

(4)
Pessimistic Asset Allocation and Choquet Utility

- **Choquet Expected Utility**: 
  \[ \mathbb{E}_\nu[u(R_p)] = \int_0^1 u\left(F_{R_p}^{-1}(\vartheta)\right) d\nu(\vartheta), \]  
  \(3\)

  \(u(\cdot)\): utility function (Hp monotone increasing), \(\nu(\cdot)\): distortion function, \(\vartheta\): probability level, \(F_{R_p}(r_p)\): distribution function of \(R_p\) at \(r_p\);

- Let \(\alpha\) be a low probability level, if \(\nu_\alpha(\vartheta) = \min\{\vartheta/\alpha, 1\}\) (Basset et al., 2004), then
  \[ \mathbb{E}_{\nu_\alpha}[u(R_p)] = \alpha^{-1} \int_0^\alpha u\left(F_{R_p}^{-1}(\vartheta)\right) d\vartheta \]  
  \(4\)

- \(\alpha\)-risk (coherent risk measure (Artzner et al., 1999)), defined as
  \[ \varrho_{\nu_\alpha}(R_p) = -\int_0^1 F_{R_p}^{-1}(\vartheta)d\nu(\vartheta) = -\alpha^{-1} \int_0^\alpha F_{R_p}^{-1}(\vartheta)d\vartheta \]  
  \(5\)
Given $\rho_\alpha(\epsilon) = \epsilon [\alpha - I(\epsilon < 0)]$, $\mu_p = \mathbb{E}[R_p]$, the error term $\epsilon$, Bassett et al. (2004) showed that

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \mathbb{E}[\rho_\alpha(\epsilon)] = \alpha (\mu_p + \varphi_{\nu\alpha}(R_p)) ;$$

(6)
Given $\rho_\alpha(\epsilon) = \epsilon [\alpha - I(\epsilon < 0)]$, $\mu_p = \mathbb{E}[R_p]$, the error term $\epsilon$, Bassett et al. (2004) showed that

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \mathbb{E}[\rho_\alpha(\epsilon)] = \alpha (\mu_p + \varrho_\alpha(R_p)); \quad (6)$$

the portfolio $\alpha$-risk is minimized through the quantile regression model (Koenker & Bassett, 1978):

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \sum_{t=1}^{T} \rho_\alpha(r_{n,t} - w_1(\alpha) r_{1,t}^* - \ldots - w_{n-1}(\alpha) r_{n-1,t}^* - \xi(\alpha)), \quad \text{s.t. } \mu_p = c; \quad (7)$$
Given $\rho_\alpha(\epsilon) = \epsilon [\alpha - I(\epsilon < 0)]$, $\mu_p = \mathbb{E}[R_p]$, the error term $\epsilon$, Bassett et al. (2004) showed that

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \mathbb{E} [\rho_\alpha(\epsilon)] = \alpha (\mu_p + \varrho_{\nu_\alpha}(R_p));$$

(6)

the portfolio $\alpha$-risk is minimized through the quantile regression model (Koenker & Bassett, 1978):

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \sum_{t=1}^{T} \rho_\alpha(r_{n,t} - w_1(\alpha)r_{1,t}^* - \ldots - w_{n-1}(\alpha)r_{n-1,t}^* - \xi(\alpha)), \quad s.t. \mu_p = c;$$

(7)

given the impact of the estimation errors for the expected returns (Brodie, 1993; Chopra & Ziemba, 1993), we refer to Model (7) without imposing constraint on $\mu_p$. 

---

**α-risk and Quantile Regression**

- Given $\rho_\alpha(\epsilon) = \epsilon [\alpha - I(\epsilon < 0)]$, $\mu_p = \mathbb{E}[R_p]$, the error term $\epsilon$, Bassett et al. (2004) showed that

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \mathbb{E} [\rho_\alpha(\epsilon)] = \alpha (\mu_p + \varrho_{\nu_\alpha}(R_p));$$

(6)

- the portfolio $\alpha$-risk is minimized through the quantile regression model (Koenker & Bassett, 1978):

$$\min_{(\xi(\alpha), w_{-n}(\alpha)) \in \mathbb{R}^n} \sum_{t=1}^{T} \rho_\alpha(r_{n,t} - w_1(\alpha)r_{1,t}^* - \ldots - w_{n-1}(\alpha)r_{n-1,t}^* - \xi(\alpha)), \quad s.t. \mu_p = c;$$

(7)

given the impact of the estimation errors for the expected returns (Brodie, 1993; Chopra & Ziemba, 1993), we refer to Model (7) without imposing constraint on $\mu_p$. 

---

Bonaccolto G.  Asset Allocation Strategies Based on Penalized Quantile Regression 5 / 25
Bassett et al. (2004) focus on $\alpha$. We consider the entire support of the response variable conditional distribution;
Bassett et al. (2004) focus on $\alpha$. We consider the entire support of the response variable conditional distribution; let $\psi$ be an high probability level, e.g. $\psi = \{0.9, 0.95\}$, then

$$
\Psi_1(R_p, \psi) = -\psi^{-1} \int_0^{\psi} F_{R_p}^{-1}(\nu) d\nu = -\mathbb{E}[R_p | R_p \leq F_{R_p}^{-1}(\psi)]; \quad (8)
$$

$$
given \bar{\nu}, the value s.t. F_{R_p}^{-1}(\bar{\nu}) = 0, the (8) can be rewritten as

$$
\Psi_1(R_p, \psi) = -\psi^{-1} \left[ \int_0^{\bar{\nu}} F_{R_p}^{-1}(\nu) d\nu + \int_{\psi}^{\bar{\nu}} F_{R_p}^{-1}(\nu) d\nu \right],
$$

from which we can build

$$
\Psi_2(R_p, \psi) = \int_{\psi}^{\bar{\nu}} F_{R_p}^{-1}(\nu) d\nu \bigg| \bigg| \int_0^{\bar{\nu}} F_{R_p}^{-1}(\nu) d\nu + \int_{\psi}^{\bar{\nu}} F_{R_p}^{-1}(\nu) d\nu \bigg|.
$$

(9)
Bassett et al. (2004) focus on $\alpha$. We consider the entire support of the response variable conditional distribution; let $\psi$ be an high probability level, e.g. $\psi = \{0.9, 0.95\}$, then

$$\Psi_1(R_p, \psi) = -\psi^{-1} \int_0^\psi F_{R_p}^{-1}(\vartheta)d\vartheta = -\mathbb{E}[R_p|R_p \leq F_{R_p}^{-1}(\psi)]; \quad (8)$$

given $\bar{\vartheta}$, the value s.t. $F_{R_p}^{-1}(\bar{\vartheta}) = 0$, the (8) can be rewritten as

$$\Psi_1(R_p, \psi) = -\psi^{-1} \left[ \int_0^\bar{\vartheta} F_{R_p}^{-1}(\vartheta)d\vartheta + \int_{\bar{\vartheta}}^\psi F_{R_p}^{-1}(\vartheta)d\vartheta \right],$$
Bassett et al. (2004) focus on \( \alpha \). We consider the entire support of the response variable conditional distribution;

let \( \psi \) be an high probability level, e.g. \( \psi = \{0.9, 0.95\} \), then

\[
\Psi_1(R_p, \psi) = -\psi^{-1} \int_0^\psi F_{R_p}^{-1}(\vartheta) d\vartheta = -\mathbb{E}[R_p | R_p \leq F_{R_p}^{-1}(\psi)]; \tag{8}
\]

given \( \bar{\vartheta} \), the value s.t. \( F_{R_p}^{-1}(\bar{\vartheta}) = 0 \), the (8) can be rewritten as

\[
\Psi_1(R_p, \psi) = -\psi^{-1} \left[ \int_0^{\bar{\vartheta}} F_{R_p}^{-1}(\vartheta) d\vartheta + \int_{\bar{\vartheta}}^\psi F_{R_p}^{-1}(\vartheta) d\vartheta \right], \text{ from which we can build}
\]

\[
\Psi_2(R_p, \psi) = \frac{\int_{\bar{\vartheta}}^\psi F_{R_p}^{-1}(\vartheta) d\vartheta}{\int_0^{\bar{\vartheta}} F_{R_p}^{-1}(\vartheta) d\vartheta}; \tag{9}
\]
Portfolio Performance as Function of the Quantiles Levels

- $\Psi_1(R_p, \psi)$ and $\Psi_2(R_p, \psi)$ are estimated as follows:

\[
\hat{\Psi}_1(r_p, \psi) = -\frac{\sum_{t=1}^{T} r_{p,t} I(r_{p,t} \leq \hat{Q}_\psi(r_p))}{\sum_{t=1}^{T} I(r_{p,t} \leq \hat{Q}_\psi(r_p))},
\]

(10)

\[
\hat{\Psi}_2(r_p, \psi) = \frac{\sum_{t=1}^{T} r_{p,t} I(0 \leq r_{p,t} \leq \hat{Q}_\psi(r_p))}{|\sum_{t=1}^{T} r_{p,t} I(r_{p,t} < 0)|},
\]

(11)

where $\hat{Q}_\psi(r_p)$ is the estimated $\psi$-th quantile of $R_p$;
Portfolio Performance as Function of the Quantiles Levels

\[ \hat{\Psi}_1(r_p, \psi) = -\frac{\sum_{t=1}^{T} r_{p,t} l \left( r_{p,t} \leq \hat{Q}_\psi(r_p) \right)}{\sum_{t=1}^{T} l \left( r_{p,t} \leq \hat{Q}_\psi(r_p) \right)}, \]  

(10)

\[ \hat{\Psi}_2(r_p, \psi) = \frac{\sum_{t=1}^{T} r_{p,t} l \left( 0 \leq r_{p,t} \leq \hat{Q}_\psi(r_p) \right)}{\left| \sum_{t=1}^{T} r_{p,t} l \left( r_{p,t} < 0 \right) \right|}, \]  

(11)

where \( \hat{Q}_\psi(r_p) \) is the estimated \( \psi \)-th quantile of \( R_p \);

- the median regression minimizes \( \mathbb{E}[|R_p - \xi(0.5)|] \). If \( \mathbb{E}[R_p] = 0 \) and \( \xi(\psi = 0.5) = 0 \), the median regression minimizes the portfolio mean absolute deviation:

\[ \text{MAD}(R_p) = \mathbb{E} \left[ |R_p - \mathbb{E}[R_p]| \right]. \]  

(12)
Simulation exercise on portfolios containing 94 assets. We test 4 different distribution: Multivariate Normal, Multivariate $t$-Student (5 d.f.), Multivariate Skew-Normal (right and left skewed). From each distribution we generate 1000 samples with dimension $500 \times 94$;
Evidences from simulated data

- Simulation exercise on portfolios containing 94 assets. We test 4 different distribution: Multivariate Normal, Multivariate $t$-Student (5 d.f.), Multivariate Skew-Normal (right and left skewed). From each distribution we generate 1000 samples with dimension $500 \times 94$;
- we compare 4 different asset allocation strategies built from the quantile regression models at $\vartheta = \{0.1, 0.5, 0.9\}$ ($QR(0.1), QR(0.5), QR(0.9)$) and from the linear regression model ($OLS$);
Evidences from simulated data

- Simulation exercise on portfolios containing 94 assets. We test 4 different distribution: Multivariate Normal, Multivariate t-Student (5 d.f.), Multivariate Skew-Normal (right and left skewed). From each distribution we generate 1000 samples with dimension $500 \times 94$;

- we compare 4 different asset allocation strategies built from the quantile regression models at $\vartheta = \{0.1, 0.5, 0.9\}$ ($QR(0.1)$, $QR(0.5)$, $QR(0.9)$) and from the linear regression model ($OLS$);

- we compare the performance, on the basis of in-sample portfolios returns, in terms of variance, mean absolute deviation, $\alpha$-risk ($\alpha = 0.1$), $\Psi_1(R_p, \psi)$ and $\Psi_2(R_p, \psi)$, at $\psi = 0.9$;
Evidences from simulated data

Figure: Multivariate Normal. A: QR(0.1), B: QR(0.5), C: QR(0.9), D: OLS.
Evidences from simulated data

Figure: Multivariate $t$-Student. A: $QR(0.1)$, B: $QR(0.5)$, C: $QR(0.9)$, D: $OLS$. 
Evidences from simulated data

Figure: Multivariate Skew-Normal (left skewed). A: $QR(0.1)$, B: $QR(0.5)$, C: $QR(0.9)$, D: $OLS$. 
Evidences from simulated data

Figure: Multivariate Skew-Normal (right skewed). A: $QR(0.1)$, B: $QR(0.5)$, C: $QR(0.9)$, D: $OLS$. 
The Inclusion of the $\ell_1$-norm Penalty

- Focus on large portfolios. Problematic issues from their dimensionality;
The Inclusion of the $\ell_1$-norm Penalty

- Focus on large portfolios. Problematic issues from their dimensionality;
- penalized quantile regression model:

$$\arg\min_{(w_{-k}(\vartheta), \xi(\vartheta)) \in \mathbb{R}^n} \sum_{t=1}^{T} \rho_{\vartheta} \left( r_{k,t} - \sum_{j \neq k} w_j(\vartheta) r_{j,t}^* - \xi(\vartheta) \right) + \lambda \sum_{j \neq k} |w_j(\vartheta)|; \quad (13)$$
The Inclusion of the $\ell_1$-norm Penalty

- Focus on large portfolios. Problematic issues from their dimensionality;
- penalized quantile regression model:

$$\arg\min_{(w_{-k}(\vartheta), \xi(\vartheta))} \left\{ \sum_{t=1}^{T} \rho_{\vartheta} \left( r_{k,t} - \sum_{j \neq k} w_j(\vartheta) r_{j,t}^* - \xi(\vartheta) \right) + \lambda \sum_{j \neq k} |w_j(\vartheta)| \right\}$$

(13)

- critical issues: choice of the response variable $R_k$, $1 \leq k \leq n$ (we select the asset with the lowest in-sample $\Psi_1(R_p, \psi)$),
The Inclusion of the $\ell_1$-norm Penalty

- Focus on large portfolios. Problematic issues from their dimensionality;
- penalized quantile regression model:

$$\arg \min_{(w_{-k}(\vartheta), \xi(\vartheta)) \in \mathbb{R}^n} \sum_{t=1}^{T} \rho_{\vartheta} \left( r_{k,t} - \sum_{j \neq k} w_j(\vartheta) r_{j,t}^* - \xi(\vartheta) \right) +$$

$$+ \lambda \sum_{j \neq k} |w_j(\vartheta)|; \quad (13)$$

- critical issues: choice of the response variable $R_k$, $1 \leq k \leq n$ (we select the asset with the lowest in-sample $\Psi_1(R_p, \psi)$), and of the optimal value of $\lambda$ (Belloni & Chernozhukov, 2011).
Empirical Set-Up

Real-world data. We take into account two different datasets, consisting of the daily returns generated by the companies included, from November 2004 to November 2014, to the baskets of the Standard & Poor’s 100 and the Standard & Poor’s 500 indices, respectively. In $S&P_{100}$ $n = 94$, in $S&P_{500}$ $n = 452$;
Empirical Set-Up

- Real-world data. We take into account two different datasets, consisting of the daily returns generated by the companies included, from November 2004 to November 2014, to the baskets of the Standard & Poor’s 100 and the Standard & Poor’s 500 indices, respectively. In S&P100 $n = 94$, in S&P500 $n = 452$;
- the attention is not restricted just on the expected (in-sample) results, but we also analyze the actual (out-of-sample) performance. Rolling analysis applied with two different window sizes, $ws = \{500, 1000\}$, and step of one day ahead;
Empirical Set-Up

- Real-world data. We take into account two different datasets, consisting of the daily returns generated by the companies included, from November 2004 to November 2014, to the baskets of the Standard & Poor’s 100 and the Standard & Poor’s 500 indices, respectively. In $S&P100$ $n = 94$, in $S&P500$ $n = 452$;
- the attention is not restricted just on the expected (in-sample) results, but we also analyze the actual (out-of-sample) performance. Rolling analysis applied with two different window sizes, $ws = \{500, 1000\}$, and step of one day ahead;
- strategies compared through the measures: mean, standard deviation, mean absolute deviation, Value-at-Risk, $\alpha$-risk, $\Psi_1(R_p, \psi)$, $\Psi_2(R_p, \psi)$, turnover and the final wealth;
Empirical Set-Up

- Real-world data. We take into account two different datasets, consisting of the daily returns generated by the companies included, from November 2004 to November 2014, to the baskets of the Standard & Poor’s 100 and the Standard & Poor’s 500 indices, respectively. In $S&P100 \ n = 94$, in $S&P500 \ n = 452$;
- the attention is not restricted just on the expected (in-sample) results, but we also analyze the actual (out-of-sample) performance. Rolling analysis applied with two different window sizes, $ws = \{500, 1000\}$, and step of one day ahead;
- strategies compared through the measures: mean, standard deviation, mean absolute deviation, Value-at-Risk, $\alpha$-risk, $\Psi_1(R_p, \psi)$, $\Psi_2(R_p, \psi)$, turnover and the final wealth;
- the in-sample results are consistent with the expectations.
Out of Sample; $S&P100, ws = 1000$

**Figure:** LASSO and $PQR(\vartheta)$: strategies from ordinary least squares and quantile regression with $\ell_1$-norm penalty.
Out of Sample; \( S&P500, ws = 1000 \)

Figure: LASSO and \( \text{PQR}(\vartheta) \): strategies from ordinary least squares and quantile regression with \( \ell_1 \)-norm penalty.
Figure: LASSO and PQR(φ): strategies from ordinary least squares and quantile regression with $\ell_1$-norm penalty.
Out of Sample; $S&P500$, $ws = 1000$

Figure: LASSO and $PQR(\vartheta)$: strategies from ordinary least squares and quantile regression with $\ell_1$-norm penalty.
Evolution of the Portfolio Value

Figure: Rolling analysis is implemented with window size of 1000 observations.
Turnover

Figure: Rolling analysis is implemented with window size of 1000 observations.
The Role of the Intercept

- From the quantile regression model, we consider the out-of-sample residual:

\[
\epsilon_{t+1}(\vartheta) = r_{k,t+1} - \left[ \hat{\xi}_t(\vartheta) + \sum_{j \neq k} \hat{w}_{j,t}(\vartheta)r_{j,t+1}^* \right]
\]  

(14)
The Role of the Intercept

- From the quantile regression model, we consider the out-of-sample residual:

\[ \epsilon_{t+1}(\vartheta) = r_{k,t+1} - \left[ \hat{\xi}_t(\vartheta) + \sum_{j \neq k} \hat{w}_{j,t}(\vartheta) r_{j,t+1}^* \right] \]  

(14)

- under the budget constraint, the portfolio return can be written as

\[ r_{p,t+1}(\vartheta) = \epsilon_{t+1}(\vartheta) + \hat{\xi}_t(\vartheta) \]  

(15)
The Role of the Intercept

- From the quantile regression model, we consider the out-of-sample residual:

\[ \epsilon_{t+1}(\vartheta) = r_{k,t+1} - \left[ \hat{\xi}_t(\vartheta) + \sum_{j \neq k} \hat{\omega}_{j,t}(\vartheta) r_{j,t+1}^* \right] \quad (14) \]

- Under the budget constraint, the portfolio return can be written as

\[ r_{p,t+1}(\vartheta) = \epsilon_{t+1}(\vartheta) + \hat{\xi}_t(\vartheta) \quad (15) \]

- High/low \( \vartheta \) levels: benefits/losses from positive/negative intercept values; low/high \( \vartheta \) levels: benefits/losses from the residuals. The opposite effects are, on average, balanced;
The Role of the Intercept

- From the quantile regression model, we consider the out-of-sample residual:

\[ \epsilon_{t+1}(\vartheta) = r_{k,t+1} - \left[ \hat{\xi}_t(\vartheta) + \sum_{j \neq k} \hat{w}_{j,t}(\vartheta)r_{j,t+1}^* \right] \] (14)

- under the budget constraint, the portfolio return can be written as

\[ r_{p,t+1}(\vartheta) = \epsilon_{t+1}(\vartheta) + \hat{\xi}_t(\vartheta) \] (15)

- high/low \( \vartheta \) levels: benefits/losses from positive/negative intercept values; low/high \( \vartheta \) levels: benefits/losses from the residuals. The opposite effects are, on average, balanced;

- the intercepts distributions have a lower dispersion with respect to the residuals distributions, with benefits in the out-of-sample performance.
Quantile regression based asset allocation corresponds to the minimization of lower tail risk, mean absolute deviation or maximization of a reward measure, depending on the quantile we are looking at. Within such an analyses we introduced a novel performance measure, that is clearly related to specific portfolio return distribution quantiles;
Concluding Remarks

- Quantile regression based asset allocation corresponds to the minimization of lower tail risk, mean absolute deviation or maximization of a reward measure, depending on the quantile we are looking at. Within such an analyses we introduced a novel performance measure, that is clearly related to specific portfolio return distribution quantiles;

- combination of quantile regression and regularization, with the $\ell_1$-norm penalty, to estimate the portfolio weights, given the potentially large cross-sectional dimension of portfolio;
Concluding Remarks

- Quantile regression based asset allocation corresponds to the minimization of lower tail risk, mean absolute deviation or maximization of a reward measure, depending on the quantile we are looking at. Within such an analyses we introduced a novel performance measure, that is clearly related to specific portfolio return distribution quantiles;

- combination of quantile regression and regularization, with the $\ell_1$-norm penalty, to estimate the portfolio weights, given the potentially large cross-sectional dimension of portfolio;

- our empirical evidences, based both on simulations and real data examples, highlight the features and the benefits of our methodological contributions;
Concluding Remarks

- Quantile regression based asset allocation corresponds to the minimization of lower tail risk, mean absolute deviation or maximization of a reward measure, depending on the quantile we are looking at. Within such an analyses we introduced a novel performance measure, that is clearly related to specific portfolio return distribution quantiles;

- combination of quantile regression and regularization, with the $\ell_1$-norm penalty, to estimate the portfolio weights, given the potentially large cross-sectional dimension of portfolio;

- our empirical evidences, based both on simulations and real data examples, highlight the features and the benefits of our methodological contributions;

- further research: inclusion of different penalty functions (e.g. non-convex) and identification of the optimal quantile level.
Main References I


THANKS FOR YOUR ATTENTION!